

THE MATH OF PATTERNS



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PROJECT SITE: <http://www.theshapeofmath.com/princeton/dynsys>

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DEPARTMENT: Independent Concentration in Quantitative Neuroscience (AB)

POSITION: Undergraduate student

The goal of this project is to provide a series of multimedia web resources that can supplement a course in dynamical systems. Motivated through several courses at Princeton and Oxford universities based on dynamical systems, differential equations, biology, and neuroscience, I sought to create a visual guide to the mathematics of pattern formation. The visual nature of the mathematics naturally lends itself to being understood with the help of short movies, animations, audio clips, and illustrations. It is my hope that the use of these materials can help students gain a stronger intuition for ideas like Turing (or diffusion-driven) instabilities, and to give students a historical and physical background for the mathematics.

Recommended Use

This website was created in such a way that it could be read from beginning to end like any set of course notes, or a chapter in a textbook. Students with a general interest in pattern formation, and some mathematical background should be able to go to the website and gain a better understanding of the mathematical, physical, and historical ideas behind models of pattern formation.

In addition, the interactive nature of the materials can be used as teaching aids in a dynamical systems course in order to supplement the lectures. The integration of the website to a classroom environment can be accomplished in several ways, for example:

- 1 Students can read through the relevant pages, so that they arrive to class with a grasp on the intuitive ideas and can understand the mathematical analysis more readily.
- 2 Flash videos can be played at the beginning of the class
- 3 The animations can be used by instructors to supplement lecture slides, and students can refer to them and listen to the audio accompaniments when they are home to refresh their memory.
- 4 The MATLAB demos and the embedded code can help students understand how the animations are created and serve as a primer to the equally interesting and fundamental field of numerical methods

For more details, please see the About (Part V) page of the project website.

Content & Organization

All the course materials can be found online at <http://www.theshapeofmath.com/princeton/dynsys>

The website contains the following multimedia components:

- 1 Written notes with a list of references at the end of each section
- 2 GIF animations with audio accompaniment, and illustrations
- 3 Short Youtube videos created using Flash
- 4 Embedded MATLAB code and short screen casts stepping through code

The content is divided into the following sections:

Part I: Introduction

- I. A historical overview of the subject and an outline of the examples used in later sections.

Part II: Reaction-Diffusion Equations

- I. Overview of reactions and diffusion separately
- II. Reaction-Diffusion models and sensitivity to spatial domain

Part III. Turing Instabilities

- I. Derivation of the conditions for Turing instabilities
- II. Example: 1D Gierer-Meinhardt model
- III. 2D models and going beyond Turing patterns

Part IV: Appendices

- I. A primer in numerical methods
- II. Links to YouTube videos

Part V: About

- I. Description of the project, author information, and acknowledgements

Acknowledgements

The Mathematics of Patterns forms a portion of my independent work for the undergraduate certificate in applied and computational mathematics with the Program in Applied and Computational Mathematics at Princeton University. The project is jointly supervised by Prof. Howard Stone (Princeton) and Dr. Philippe Trinh (Oxford). I also thank Prof. Philip Holmes for suggesting this project for the DSWeb 2013 contest.

Please note that the main website on which this project is hosted belongs to Dr. Philippe Trinh, who was kind enough to let me use his website to place my materials.